Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number

Candidate Number

Sample Assessment Materials

(Time: 1 hour 30 minutes)

Paper Reference **9FM0/3C**

Further Mathematics

Advanced

Paper 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1/1/1/1/

Pearson

Year 1 Work, Energy and Power - Inclined planes, Variable resistance

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\mathrm{m\,s^{-2}}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A van of mass 750 kg is moving along a straight horizontal road.

At the instant when the speed of the van is $v\,\mathrm{m\,s^{-1}}$, the resistance to the motion of the van is modelled as a force of magnitude $(200 + v^2)$ N.

When the engine of the van is working at a constant rate of 12 kW, and the van is moving at a constant speed,

(a) show that the van must be moving at $20\,\mathrm{m\,s^{-1}}$, justifying your answer.

(4)

Later on, the van is moving up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{15}$

At the instant when the speed of the van is $v\,\mathrm{m\,s^{-1}}$, the resistance to the motion of the van from non-gravitational forces is modelled as a force of magnitude $(200 + v^2)$ N.

The engine of the van is now working at a constant rate of 15 kW.

(b) Find the acceleration of the van at the instant when $v = 10$

(5)

let's illustrate the information above in a detailed diagram - label the weight, the resistance to motion $(200+v^2)$ and the power rearranged

formula: $P = Fv$ ← velocity in $\mathrm{ms^{-1}}$

Power in WATTS; Force in NEWTONS

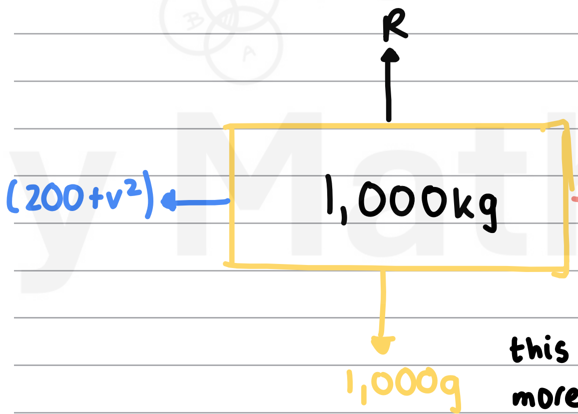
$F = \frac{P}{v}$

...where:

12kW $\xrightarrow[\text{to Watts}]{\text{convert}}$ 12,000W

and $v = V$

↳ NOTE: question is asking us to show that $v = 20\mathrm{ms^{-1}}$ so let's consider v as unknown and then eventually solve/sub for $v = 20\mathrm{ms^{-1}}$



$F = \frac{12{,}000}{v}$

NOTE: could've done this as a separate line of working but it's more efficient in the exam to write in the FORCE as the POWER rearranged

now use the fact that the car is moving at a constant speed - this means that it's in non-stationary equilibrium, so forces left = forces right

$\Rightarrow \frac{12{,}000}{v} = 200 + v^2$ $\quad \times v \quad \times v$ $\Big\}$ $v^3 + 200v - 12{,}000 = 0$

now we can do one of two methods to show $v = 20\,ms^{-1}$

WAY 1: use substitution

if subbing in $v = 20\,ms^{-1}$ into the cubic gives '0' - this means that it must be the only solution

LHS: $(20)^3 + 200(20) - 12{,}000$ RHS: $= 0$

$= 8{,}000 + 4{,}000 - 12{,}000$

$= 12{,}000 - 12{,}000$

$= 0 \implies LHS = RHS$

$\boxed{\therefore v = 20}$

WAY 2: use factor theorem

know from Core Pure Yr 1 that the best way to solve a cubic for v is to use factor theorem: this states that if $(x - \alpha)$ is a factor, then $f(\alpha) = 0$ - hence if $v = 20$ was a factor, then $f(-20)$ would be a factor - let's prove however why it has to be the only factor as well ...using polynomial division on the cubic:

$$
\begin{array}{r|l}
 & v^2 + 20v + 600 \\
\hline
v - 20 & v^3 + 0v^2 + 200v - 12{,}000 \\
 & -\ \underline{v^3 - 20v^2} \\
 & \quad 20v^2 + 200v \\
 & -\ \underline{20v^2 - 400v} \\
 & \qquad 600v - 12{,}000 \\
 & -\ \underline{600v - 12{,}000} \\
 & \qquad\qquad 0
\end{array}
$$

$\boxed{\therefore v = 20\,ms^{-1}}$

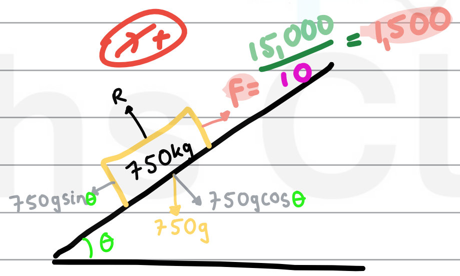
(b) now the van is moving up the INCLINED PLANE - illustrating this diagrammatically: label the weight resolved, the resistance to motion, and the power rearranged

formula: $P = Fv$

$\implies F = P/v$

...where:

$15kW \xrightarrow{\times 1000} 15{,}000W$ convert to watts



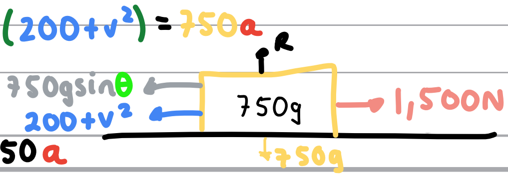
...to find the acceleration let's resolve horizontally - subbing into Neuton's Second Law:

turn axis

formula: $\Sigma F_x = ma$

$\Sigma F_x = 1{,}500 - 750g\sin\theta - (200 + v^2) = 750a$

sub in $\sin\theta = 1/15$ (given in question)



$1{,}500 - 750g(1/15) - 300 = 750a$

expand and solve for 'a'

$\Rightarrow 710 = 750a$

$\div 750 \quad \div 750$

$a = \frac{710}{750} \approx 0.9466.. \approx \boxed{0.947\,ms^{-1}\,(3sf)}$

Year 2 Elastic Strings and Springs; conservation of mechanical energy, friction, Inclined planes

2.

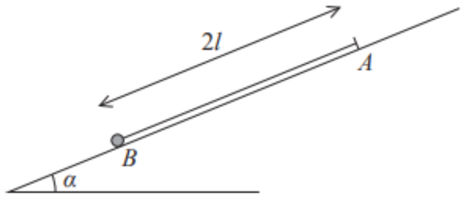


Figure 1

One end of a light elastic string, of natural length l and modulus of elasticity $\frac{3}{4}mg$, is attached to a particle of mass m. The other end of the string is attached to a fixed point A on a rough inclined plane. The plane is inclined at angle α to the horizontal, where

$\tan\alpha = \frac{5}{12}$

Initially the particle is held at the point B on the plane, where $AB = 2l$ and B lies below A on the line of greatest slope through A, as shown in Figure 1.

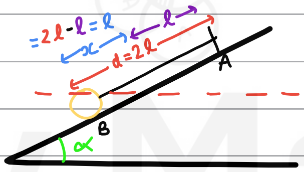
The particle is released from rest at B and first comes to instantaneous rest at the point C, where C is between A and B and $AC = \frac{8}{5}l$.

Find the coefficient of friction between the particle and the plane.

(6)

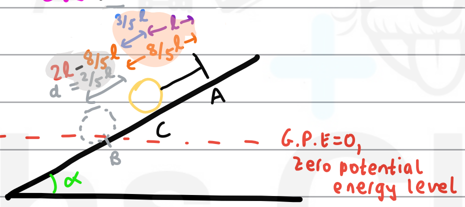
the most important thing to consider when approaching elastic strings and springs questions is drawing a detailed diagram - here, let's do one for BEFORE and one for AFTER the particle travels up the plane - label respective energies

BEFORE:



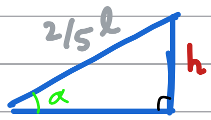
· E.P.E

AFTER:



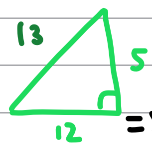
· E.P.E

· G.P.E - but need perp. distance, call it 'h', travelled up plane - hence deriving the following triangle from the diagram and using trig



$\Rightarrow h = \frac{2}{5}l\sin\alpha$

and given $\tan\alpha = 5/12$ - using Pythag. triple: 5, 12, 13

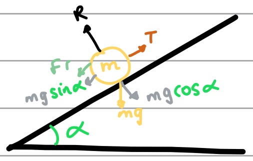


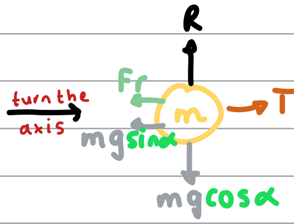
$\Rightarrow \sin\alpha = \frac{O}{H}$
$= 5/13$

$\Rightarrow \cos\alpha = A/H$
$= 12/13$

∴ subbing in $\sin\alpha = 5/13$ into expression for 'h'

$\Rightarrow h = \frac{2}{5}l\left(\frac{5}{13}\right) = \frac{2}{13}l$





w.d by friction - need formula for friction

formula: $\underbrace{Fr}_{\text{FRICTION}} = \underbrace{\mu}_{\text{COEFFICIENT OF FRICTION}} R$ ← REACTION FORCE - need to resolve vertically ∴ need FORCE DIAGRAM

this is what we're looking for!

$R(\uparrow)$: $R = mg\cos\alpha$ - subbing in $\cos\alpha = 12/13$

$\Rightarrow R = \frac{12}{13}mg$ $\quad \therefore Fr = \mu\left(\frac{12}{13}mg\right)$

sub above into the work-energy principle: includes dissipative forces)

$$\underbrace{\text{w.d in}}_{n/a} + \underbrace{K.E_i}_{\text{kinetic initial}} + \underbrace{G.P.E_i}_{\text{gravitational potential initial}} + \underbrace{E.P.E_i}_{\text{elastic potential initial}} = \underbrace{K.E_f}_{\text{kinetic energy final}} + \underbrace{G.P.E_f}_{\text{gravitational potential final}} + \underbrace{E.P.E_f}_{\text{elastic potential final}} + \text{w.d against friction}$$

$$\frac{1}{2}mu^2 + mgh_1 + \frac{\lambda x^2}{2l} = \frac{1}{2}mv^2 + mgh_2 + \frac{\lambda x^2}{2l} + Fr \times d$$

subbing into above

$$0 + 0 + \frac{\frac{3}{4}\cancel{mg}(l)^2}{2l} = 0 + \frac{2}{13}l\cancel{mg} + \frac{\frac{3}{4}\cancel{mg}\left(\frac{3}{5}l\right)^2}{2l} + \frac{12}{13}\cancel{mg}\mu\left(\frac{2}{5}\right)$$

cancel mg's and expand out brackets

$$\frac{3}{8}l = \frac{2}{13}l + \frac{27}{200}l + \mu\frac{24}{65}$$

collect like 'l' terms:

$$\frac{28}{325}l = \frac{24}{65}\mu$$

$$\div\frac{24}{65} \qquad \div\frac{24}{65}$$

$$\boxed{\mu = \frac{7}{30}\ (0.233\ldots)}$$

Year 2 Vector Momentum and Impulse; finding angle of deflection

3. A particle, P, of mass 3 kg is moving with velocity $(2\mathbf{i} + \mathbf{j})\,\mathrm{m\,s^{-1}}$ when it receives an impulse $\mathbf{I}$ of magnitude $\sqrt{130}$ N s. Immediately after receiving the impulse, P is moving with velocity $(-\mathbf{i} + \lambda\mathbf{j})\,\mathrm{m\,s^{-1}}$, where λ is a positive constant.

(a) Find $\mathbf{I}$, giving your answer in terms of $\mathbf{i}$ and $\mathbf{j}$.

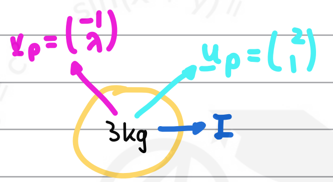
(6)

The angle between the direction of motion of P immediately before receiving the impulse and the direction of motion of P immediately after receiving the impulse is $\theta°$

(b) Find the value of θ

(3)

(a) let's illustrate the above information on a detailed diagram - label the velocity before and the velocity after



we are asked to find the vector components of I, given the value of its magnitude - hence using the vector form of the Impulse-Momentum principle:

formula: $\underline{I} = m(\underline{v} - \underline{u})$

$$\begin{pmatrix}a\\ b\end{pmatrix} = 3\left(\begin{pmatrix}-1\\ \lambda\end{pmatrix} - \begin{pmatrix}2\\ 1\end{pmatrix}\right)$$

$$\Rightarrow \begin{pmatrix}a\\ b\end{pmatrix} = 3\begin{pmatrix}-3\\ \lambda-1\end{pmatrix}$$

expand into the bracket

$$\begin{pmatrix}a\\ b\end{pmatrix} = \begin{pmatrix}-9\\ 3\lambda-3\end{pmatrix}$$

and using the magnitude $= \sqrt{130}$, hence Pythagorise $\begin{pmatrix}a\\ b\end{pmatrix}$

$$\sqrt{(-9)^2 + (3\lambda-3)^2} = \sqrt{130}$$

expand within the square root:

$$\sqrt{9\lambda^2 - 18\lambda + 90} = \sqrt{130}$$

square both sides and solve for 'λ'

$$\Rightarrow 9\lambda^2 - 18\lambda - 40 = 0$$

solve this using your calc equtn solver

$$\Rightarrow \lambda = 10/3$$

$\therefore$ subbing into our expression for $\begin{pmatrix}a\\ b\end{pmatrix}$

$$\Rightarrow I = \begin{pmatrix}a\\ b\end{pmatrix} = \begin{pmatrix}-9\\ 3(10/3)-3\end{pmatrix} = \boxed{\begin{pmatrix}-9\\ 7\end{pmatrix}\text{Ns}}$$

(b) we know 'λ' from (a), so can sub it in and find the angle of deflection between $u_p = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $v_p = \begin{pmatrix} -1 \\ 10/3 \end{pmatrix}$ - there are two ways to do this

WAY 1: using the scalar product

formula: $\cos\theta = \dfrac{a \cdot b}{|a||b|}$ ← scalar product / ← product of magnitudes

subbing into the above

$$\cos\theta = \frac{\begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 10/3 \end{pmatrix}}{\sqrt{(2)^2 + (1)^2}\sqrt{(-1)^2 + (10/3)^2}}$$

and evaluate the scalar product and the magnitude

$$\cos\theta = \frac{-2 + 10/3}{\sqrt{5}\sqrt{109/9}} = \frac{4/3}{\frac{\sqrt{545}}{3}}$$

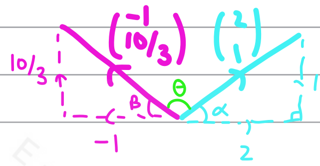
$$\Rightarrow \cos\theta = 4/\sqrt{545}$$

$$\therefore \theta = \cos^{-1}\left(4/\sqrt{545}\right)$$

$$= 80.13419\ldots$$

$$\boxed{= 80.1^\circ \text{ (3 s.f)}}$$

WAY 2: using trig and vector triangles



see from the diagram that angle θ is obtained from

$$\theta = 180^\circ - \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{10}{3}\right)$$

↳ NOTE: consider the vectors as side lengths (∴ ignore -ves)

$$\Rightarrow 180^\circ - 26.5650\ldots - 73.3007\ldots$$

$$= 80.13419\ldots$$

$$\boxed{= 80.1^\circ \text{ (3 s.f)}}$$

Year 2 Elastic Strings and Springs; equilibrium problem, dynamic problem, conservation of mechanical energ

4.

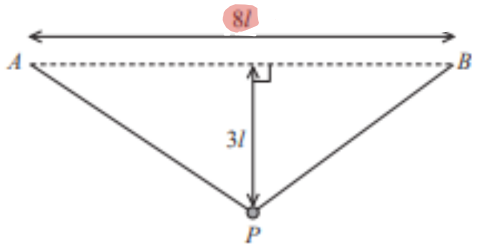


Figure 2

A light elastic string, of natural length $8l$ and modulus of elasticity kmg, has its ends attached to two points A and B, where $AB = 8l$ and AB is horizontal.

A pebble, P, of mass m is attached to the midpoint of the string. The pebble rests in equilibrium at a distance $3l$ vertically below AB, as shown in Figure 2. The pebble is modelled as a particle, and air resistance is modelled as negligible.

(a) Show that $k = \frac{10}{3}$ **(4)**

The pebble is pulled vertically downwards from its equilibrium position until the total length of the string is $\frac{40}{3}l$. The pebble is released from rest.

(b) Find the acceleration of P at the instant it is released from rest. **(3)**

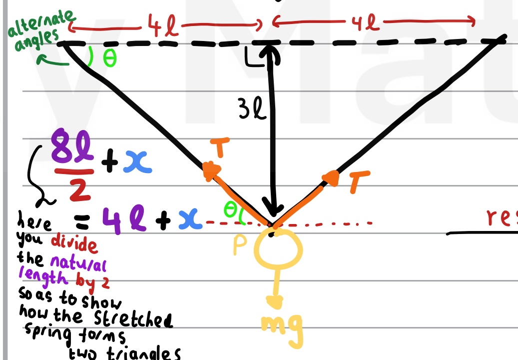
At the instant the pebble crosses the line AB, the pebble has speed v.

(c) Find v. **(3)**

In an experiment, when the natural length of the string was 2 m, it was found that the speed of P at the instant when it crossed the line AB was $1.5\,\text{m s}^{-1}$.

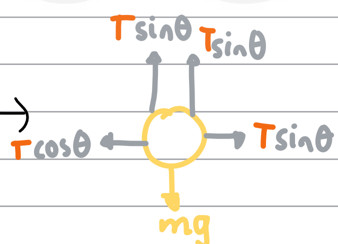
(d) Considering the model, suggest a reason, other than air resistance, why the model and the experiment give different values. **(1)**

(a) first part of this elastic strings and springs question is an EQUILIBRIUM question - hence redrawing Fig 2 as a force diagram - label the weight, tension AND natural length



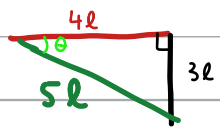
$\frac{8l}{2} + x = 4l + x$ here you divide the natural length by 2 so as to show how the stretched spring forms two triangles

resolve forces



resolving (↑): equilibrium means forces up = forces down: $2T\sin\theta = mg$

to get $\sin\theta$, it'll be useful to exploit triangle properties - use the Pythag. triple: 3, 4, 5



$\Rightarrow \sin\theta = O/H = 3/5$

$\Rightarrow \cos\theta = A/H = 4/5$

subbing $\sin\theta = 3/5$ into our equation

$$2T(3/5) = mg$$
$$\Rightarrow \tfrac{6}{5}T = mg$$
$$\div 6/5 \qquad \div 6/5$$
$$T = \tfrac{5}{6}mg$$

now to get the modulus of elasticity, 'k'mg, can sub into formula for strings and springs (on one triangle)

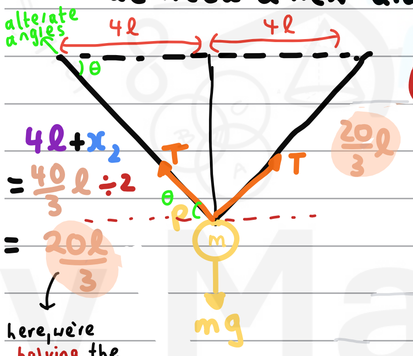
formula: $T = \dfrac{\lambda x}{l}$ (λ: MOD. OF ELASTICITY; x: EXTENSION in m; l: NATURAL LENGTH in m)

subbing in-into from one triangle:
$$\tfrac{5}{6}mg = \frac{kmg(\cancel{l})}{4\cancel{l}}$$

cancel mg's

$$\Rightarrow \frac{k}{4} = 5/6$$
$$\boxed{\therefore k = 20/6 = 10/3}$$

(b) now we have a dynamics problem that involves elastic strings and springs
→ we need a new diagram for the new extension

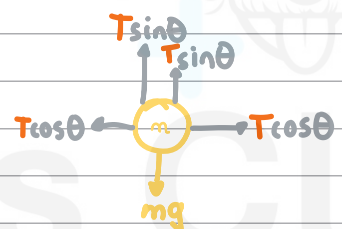


$4l + x_2$
$= \frac{40}{3}l \div 2$
$= \frac{20l}{3}$

here, we're halving the total length of the string as want to consider each triangle separately

(↑+)

resolving i-j components →



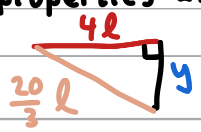
and using Newton's 2nd law to get 'a'

formula: $\Sigma F_x = ma$

subbing into above

$$2T\sin\theta - mg = ma$$ ← what we're trying to solve for!

... and we can get $\sin\theta$ by exploiting our triangle properties with trig again:



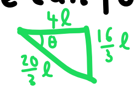
but first let's do Pythagoras' to get the unknown vertical length 'y'

$$\left(\tfrac{20}{3}l\right)^2 - (4l)^2 = y^2$$

expand out the brackets

$$\frac{256}{9}l^2 = y^2$$
$$\Rightarrow y = \frac{16}{3}l$$

∴ now we can form a trig triangle.



$\Rightarrow \sin\theta = O/H = \frac{16}{3}\ell \Big/ \frac{20}{3}\ell = \frac{4}{5}$

$\cos\theta = A/H = \frac{4\ell}{\frac{20}{3}\ell} = \frac{12}{20} = 3/5$

$\therefore \sin\theta = 4/5$

... and can get the value for T using our formula for tension (on one triangle):

formula: $T = \frac{\lambda x}{\ell}$ } subbing in: $T = \frac{\frac{10}{3}mg\left(\frac{8}{3}\ell\right)}{4\ell}$

expand

$= \frac{20}{9}mg$

NOW subbing in the value for $\sin\theta$ AND the value for T into our new equation

$\Rightarrow 2\left(\frac{20}{9}mg\right)\left(4/5\right) - mg = ma$

expand above

$\frac{32}{9}g - g = a$

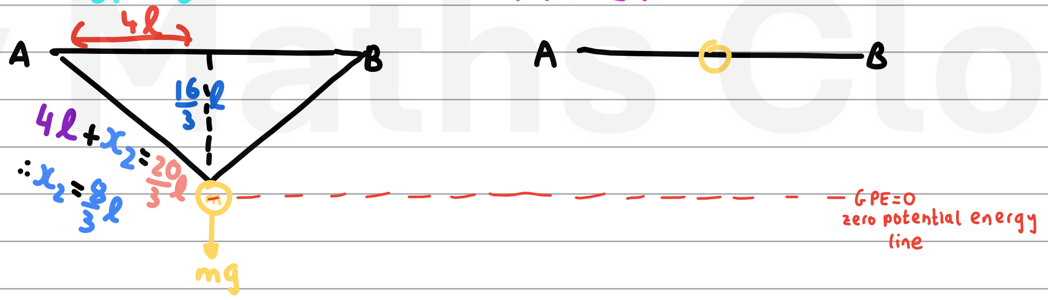
collect like terms

$\Rightarrow \boxed{a = \frac{23}{9}g \ ms^{-1}}$

(c) recognise that the fact we need to find 'v' hints at the need for us to use the work-energy principle: means momentum before = momentum after

↳ drawing two diagrams for this: one for BEFORE and one for AFTER the particle crosses the line AB (and labelling respective energies)

BEFORE: AFTER:



• E.P.E

• K.E

• G.P.E

sub above into the work-energy principle : includes dissipative forces)

$$\underset{n/a}{W.d\ in} + \underset{\text{kinetic initial}}{K.E_i} + \underset{\text{gravitational potential initial}}{G.P.E_i} + \underset{\text{elastic potential initial}}{E.P.E_i} = \underset{\text{kinetic energy final}}{K.E_f} + \underset{\text{gravitational potential final}}{G.P.E_f} + \underset{\text{elastic potential final}}{E.P.E_f} + W.d \text{ against friction}$$

$$\frac{1}{2}mu^2 + mgh_1 + \frac{\lambda x^2}{2l} = \frac{1}{2}mv^2 + mgh_2 + \frac{\lambda x^2}{2l} + Fr \times d$$

subbing into above (now treating string AS A WHOLE rather than splitting it into two triangles)

need to find!

$$0 + 0 + 2 \times \frac{\frac{10}{3}mg\left(\frac{8}{3}l\right)^2}{2(4l)} = \frac{1}{2}mv^2 + mg\left(\frac{16}{3}l\right)$$

cancel m's and simplify

$$\frac{160gl}{27} - \frac{16}{3}gl = \frac{1}{2}v^2$$

$$\Rightarrow \frac{1}{2}v^2 = \frac{16}{27}gl$$

$$\Rightarrow v^2 = \frac{32}{27}gl$$

$$\boxed{\Rightarrow v = \sqrt{\frac{32}{27}gl}\ ms^{-1}}$$

5. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*]

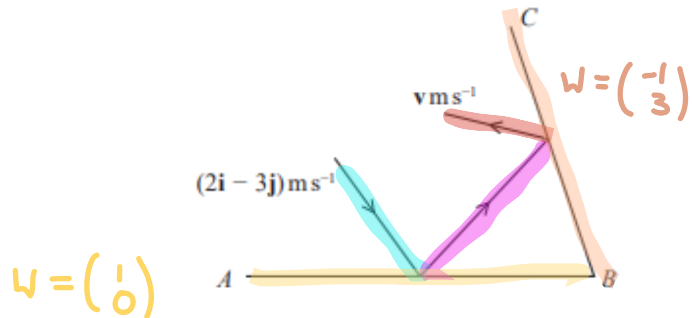


Figure 3

Figure 3 represents the plan view of part of a horizontal floor, where AB and BC represent fixed vertical walls. The direction of $\overrightarrow{AB}$ is in the direction of the vector **i** and the direction of $\overrightarrow{BC}$ is in the direction of the vector $(-\mathbf{i} + 3\mathbf{j})$.

A small ball is projected along the floor towards wall AB so that, immediately before hitting wall AB, the velocity of the ball is $(2\mathbf{i} - 3\mathbf{j})\,\mathrm{m\,s^{-1}}$.

The ball hits wall AB and then hits wall BC.

The coefficient of restitution between the ball and wall AB is $\frac{1}{2}$

The coefficient of restitution between the ball and wall BC is $\frac{1}{3}$

The velocity of the ball immediately after hitting wall BC is $\mathbf{v}\,\mathrm{m\,s^{-1}}$

The floor and the walls are modelled as being smooth. The ball is modelled as a particle.

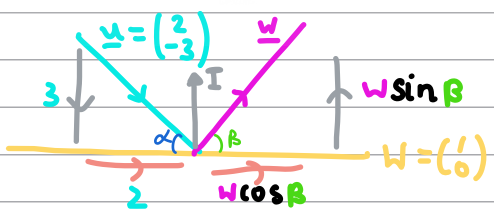
Show that $\mathbf{v} = \left(-\mathbf{i} + \frac{1}{2}\mathbf{j}\right)$.

(12)

here instantly we can see that we have a successive oblique collisions with VECTOR WALLS question - let's tackle each collision separately

...first just the collision of the ball with AB:

... diagram:



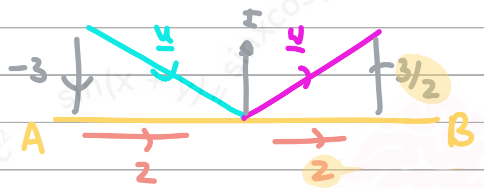
we need to get the vector components of $\underline{w}$

... perp. component: IMPULSE acts perpendicular to the surface of impact ∴ only the perp. comps will change - apply NEL rearranged (multiply by $e = \frac{1}{2}$)

$w\sin\beta = \frac{1}{2} \times -3 = -\frac{3}{2}$

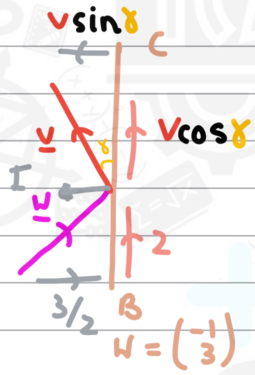
...parallel components:
no change as IMPULSE doesn't act in this direction
$\Rightarrow w\cos\beta = 2$

...putting above on a diagram:



$\Rightarrow \underline{w} = \begin{pmatrix} 2 \\ 3/2 \end{pmatrix}$

...now let's focus on the collision of the ball with wall BC
↳ notice this is a vector wall ∴ rotating it so it's vertical in the diagram:



using our two formulae for collisions with vector walls:

formula: $\underline{u}\cdot\underline{W} = \underline{v}\cdot\underline{W}$
↳ vector wall

subbing into the above:

$\begin{pmatrix} 2 \\ 3/2 \end{pmatrix}\cdot\begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}\cdot\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

evaluating scalar product

$-2 + 9/2 = -a + 3b$

$\Rightarrow -a + 3b = 5/2$ -①

formula: $-e\underline{u}\cdot\underline{I} = \underline{v}\cdot\underline{I}$
↳ impulse

but need the IMPULSE i.e the vector perpendicular to the surface of impact - can find this in one of three ways

WAY 1: using the scalar product	WAY 2: as linear equtns	WAY 3: 90° rotation
know that for perpendicular	$\begin{pmatrix} -1 \\ 3 \end{pmatrix} \therefore m_1 = \frac{3}{-1} = -3$ $\Rightarrow y = -3x$	treating the perp. vector as a 90° rotation of $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

vectors $\underline{a}\cdot\underline{b}=0$

$\Rightarrow \begin{pmatrix}-1\\3\end{pmatrix}\cdot \underline{I} = 0$

$\therefore$ by inspection,

$\underline{I} = \begin{pmatrix}3\\1\end{pmatrix}$

or $\underline{I} = \begin{pmatrix}-3\\-1\end{pmatrix}$

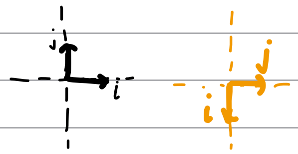
$\therefore$ formula: $m_1 \times m_2 = -1$

$\Rightarrow m_2 = $ -ve reciprocal of $m_1 \therefore 1/3$

has a vector: $\begin{pmatrix}3\\1\end{pmatrix}$

or any scalar multiple: $\begin{pmatrix}-3\\-1\end{pmatrix}$

... clockwise:



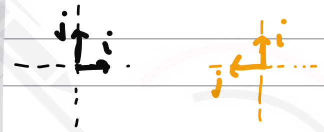
$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ $\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$

$\therefore$ as a linear transformation.

(using $Mx = y$)

$\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}\begin{pmatrix}-1\\3\end{pmatrix} = \begin{pmatrix}3\\1\end{pmatrix}$

... anticlockwise:



$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$

$\therefore$ as a linear transformation:

(using $Mx = y$)

$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\begin{pmatrix}-1\\3\end{pmatrix} = \begin{pmatrix}-3\\-1\end{pmatrix}$

$\therefore \underline{I} = \begin{pmatrix}3\\1\end{pmatrix}$ or $\begin{pmatrix}-3\\-1\end{pmatrix}$

... using $\begin{pmatrix}3\\1\end{pmatrix}$ and subbing into the formula:

$-\frac{1}{3}\begin{pmatrix}2\\3/2\end{pmatrix}\cdot\begin{pmatrix}3\\1\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}\cdot\begin{pmatrix}3\\1\end{pmatrix}$

evaluating above

$-\frac{1}{3}\left(\frac{15}{2}\right) = 3a + b$

$\Rightarrow 3a + b = -5/2$ — ②

solving ① and ② simultaneously

① − 3 × ②

$-a + 3b = 5/2$

$- \quad 9a + 3b = -15/2$

$-10a = 10$ $\div -10$

$\Rightarrow a = -1$

subbing into ① or ②

... into ①:

$-(-1) + 3b = 5/2$

$\Rightarrow 3b = 3/2$ $\div 3$

$\Rightarrow b = 1/2$

$\boxed{\therefore \underline{v} = \begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}-1\\1/2\end{pmatrix}}$

Year 1 Elastic Collisions in 1D; properties of Coefficient of restitution, kinetic energy

6. A particle, P, of mass $4m$ is moving along a straight line on a smooth horizontal plane.

A particle, Q, of mass $3m$ is at rest on the plane on the same straight line.

Particle P collides directly with particle Q.

Immediately before the collision the speed of P is ku, where k is a constant.

Immediately after the collision the speed of P is u and the speed of Q is $\frac{3u}{2}$

The coefficient of restitution between P and Q is e.

(a) (i) Show that there is only one possible value of k.

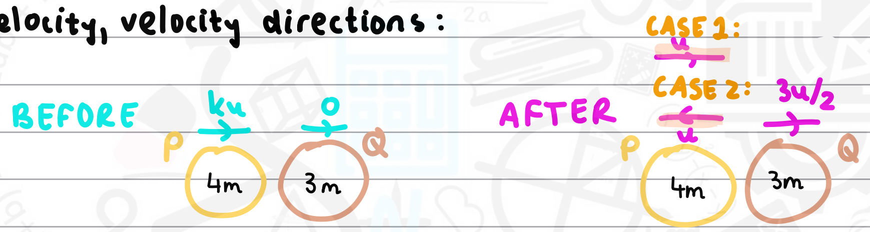
(ii) State the value of k and the value of e.

(11)

(b) Find the total kinetic energy lost in the collision between P and Q.

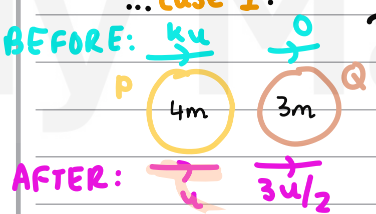
(3)

(a) realising that this is an elastic collisions in 1D question - hence showing the above information with a correct diagram - label the respective masses, velocity, velocity directions:



see above the two possible cases for direction of motion of P AFTER - have to evaluate both PCLM and NEL for both cases to see why there's only one possible case for 'k'

...case 1:



evaluate at PCLM: means momentum before = momentum after:

formula: $u_P m_P + u_Q m_Q = v_P m_P + v_Q m_Q$

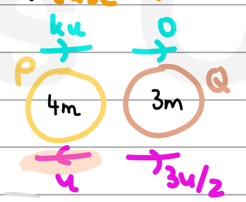
subbing into above

$k\cancel{u}(4\cancel{m}) = \cancel{u}(4\cancel{m}) + \frac{3\cancel{u}}{2}(3\cancel{m})$

cancel m's and u's

$4k = 4 + 9/2$

...case 2:



now evaluate this at PCLM: means momentum before = momentum after

—||—

subbing into formula:

$k\cancel{u}(4\cancel{m}) = (-\cancel{u})(4\cancel{m}) + (3\cancel{u}/2)(3\cancel{m})$

cancel m's and u's

$\Rightarrow 4k = -4 + 9/2$

$\Rightarrow 4k = 17/2$ ($\div 4$, $\div 4$)

$k = 17/8$

$\Rightarrow 4k = 1/2$ ($\div 4$, $\div 4$)

$k = 1/8$

now sub into NEL (Newton's Impact Law) –
perhaps use properties of 'e' to eliminate one solution

formula: $e = \dfrac{\text{speed of separation}}{\text{speed of approach}} = \dfrac{v_Q - v_P}{u_P - u_Q}$

sub into above

$\Rightarrow e = \dfrac{\frac{3}{2}u - u}{ku - 0} = \dfrac{\frac{1}{2}\cancel{u}}{k\cancel{u}}$

$\Rightarrow e = \dfrac{1}{2k}$

CASE 1: $k = 17/8$

subbing this into our value for 'e'

$e = \dfrac{1}{2(17/8)}$

$\Rightarrow e = 4/17$

CASE 2: $k = 1/8$

subbing this into our value for 'e':

$e = \dfrac{1}{2(1/8)}$

$\Rightarrow e = 4$

but know that $0 \le e \le 1$,
$\therefore$ elim. CASE 2 ($k = 1/8$)

$\Rightarrow \boxed{k = 17/8}$

(ii) $\boxed{e = 4/17}$

(b) loss of K.E implies $K.E_{initial} - K.E_{final}$, where $K.E = \frac{1}{2}mv^2$

... initially: only P has velocity (Q is at rest), so subbing into our formula for K.E

$K.E_{initial}: \frac{1}{2}(4m)\left(\frac{17}{8}u\right)^2$

$= \dfrac{289}{2}mu^2$

.finally both have K.E $\therefore$ subbing into our formula for K.E:

$K.E_{final}: \frac{1}{2}(4m)(u)^2 + \frac{1}{2}(3m)\left(\frac{3u}{2}\right)^2$

$= 2mu^2 + \frac{27}{8}mu^2 = \frac{43}{8}mu^2$

$\therefore E.K_{lost} = \dfrac{289}{32}mu^2 - \dfrac{43}{8}mu^2 = \boxed{\dfrac{117}{32}mu^2 \text{ J}}$

Year 2 Oblique Collisions; oblique Collisions with spheres, impulse

7.

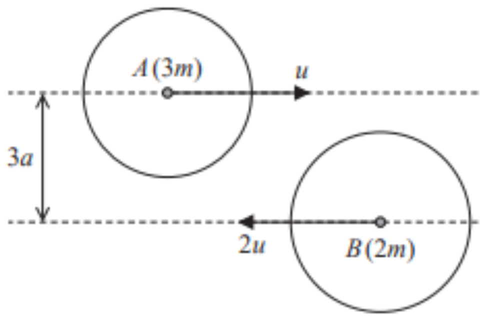


Figure 4

Two smooth uniform spheres, A and B, are moving with speeds u and $2u$ respectively on a smooth horizontal surface.

Sphere A has mass $3m$ and radius $2a$. Sphere B has mass $2m$ and radius $2a$.

The centres of the spheres are moving towards each other on parallel paths. The paths are at a distance $3a$ apart, as shown in Figure 4.

The spheres collide. The coefficient of restitution between A and B is $\frac{1}{3}$

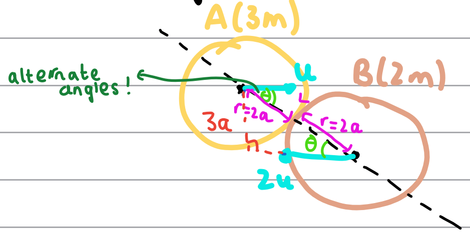
(a) Show that the magnitude of the impulse received by A in the collision is $\frac{6\sqrt{7}}{5}mu$. **(10)**

(b) Find the speed of A immediately after the collision. **(3)**

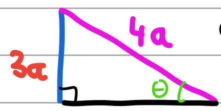
(c) State how you have used the fact that the spheres are smooth when considering their collision. **(1)**

(a) notice we're dealing with an oblique collision between two spheres question - but the spheres are moving on parallel paths rather than directly towards each other ∴ let's exploit basic geometrical properties to find the angle at which the the spheres are moving at (always measured to the line of centres)

∴ diagram:



→ See can extract the following triangle:



and exploit trig:

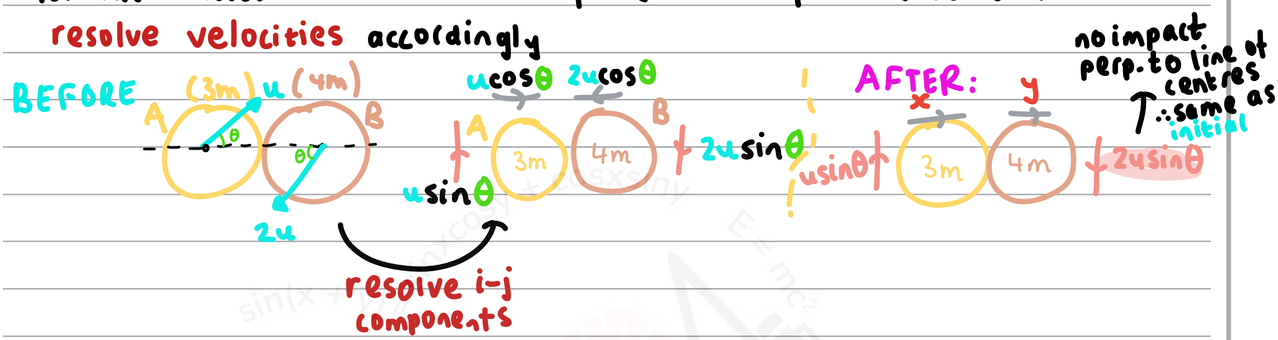
$\sin\theta = \frac{O}{H} = 3/4$

∴ can derive $\cos\theta$ using trig identity $\sin^2\theta + \cos^2\theta \equiv 1$ rearranged:

$\cos\theta = \sqrt{1 - \sin^2\theta}$

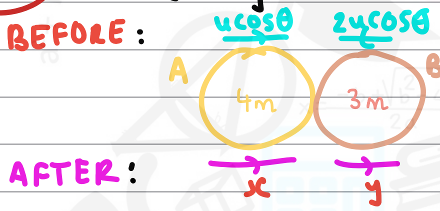
$$\Rightarrow \cos\theta = \sqrt{1-(3/4)^2}$$
$$= \sqrt{7/16} = \sqrt{7}/4$$

so now that we've exploited triangle properties, we can draw a normal collision where the line of centres is parallel to 'i' and resolve velocities accordingly



...parallel to line of centres:

(→+) treating this as a standard elastic collisions in 1D question



∴ using PCLM: momentum before = momentum after

formula: $u_A m_A + u_B m_B = v_A m_A + v_B m_B$

$4\cancel{m}u\cos\theta + 3\cancel{m}(-2u\cos\theta) = 4\cancel{m}x + 3\cancel{m}y$

cancel m's and expand:

$3x + 2y = 3u\cos\theta - 4u\cos\theta$

$\Rightarrow 3x + 2y = -u\cos\theta$ — ①

...next NEL (Newton's Impact Law):

$$\frac{1}{3} = \frac{y-x}{3u\cos\theta}$$

$\times 3u\cos\theta$ $\times 3u\cos\theta$

$\Rightarrow y - x = u\cos\theta$ — ②

solving ① and ② simultaneously:

2×② − ①

$$\begin{array}{r} 3x + 2y = -u\cos\theta \\ - \quad -2x + 2y = 2u\cos\theta \\ \hline 5x = -3u\cos\theta \end{array}$$

$\div 5$ $\div 5$

$\Rightarrow x = -\frac{3}{5}u\cos\theta$

$$\Rightarrow \underline{v}_A = \begin{pmatrix} -3/5\, u\cos\theta \\ u\sin\theta \end{pmatrix}$$

but the question asks us for the impulse ∴ subbing into our Impulse-momentum principle :

formula : $I = m(v-u)$

↳ but recognise impulse acts only along the line of centres ∴ using parallel comps of $\underline{u}_A$ and $\underline{v}_A$

$$I = 3m\left(-\tfrac{3}{5}u\cos\theta - u\cos\theta\right)$$

$$= 3m\left(-\tfrac{8}{5}u\cos\theta\right)$$

$$\Rightarrow I = -\tfrac{24}{5}mu\cos\theta$$

but know $\cos\theta = \sqrt{7}/4$ - subbing this in

$$I = -\tfrac{24}{5}mu\left(\tfrac{\sqrt{7}}{4}\right)$$

$$\Rightarrow I = -\frac{\overset{6}{\cancel{24}}\sqrt{7}}{\underset{5}{\cancel{20}}}mu$$

but asked for the magnitude

$$\boxed{\therefore I = \frac{6}{5}\sqrt{7}mu \text{ Ns}}$$

(b) to find the speed of $\underline{v}_A$ need to Pythagorise $\begin{pmatrix} -\frac{3}{5}u\cos\theta \\ u\sin\theta \end{pmatrix}$ - but sub in $\sin\theta = 3/4$ and $\cos\theta = \sqrt{7}/4$

$$\underline{v}_A = \begin{pmatrix} -\frac{3}{5}u\left(\frac{\sqrt{7}}{4}\right) \\ u\left(\frac{3}{4}\right) \end{pmatrix}$$

expanding the brackets

$$\underline{v}_A = \begin{pmatrix} -\frac{3\sqrt{7}}{20}u \\ \frac{3}{4}u \end{pmatrix}$$

Pythagorise velocity to get the speed

$$|\underline{v}_A| = \sqrt{\left(-\frac{3\sqrt{7}}{20}u\right)^2 + \left(\frac{3}{4}u\right)^2}$$

$$= \sqrt{\frac{63}{400}u^2 + \frac{9}{16}u^2} = \sqrt{\frac{18}{25}u^2}$$

$$= \boxed{\frac{3\sqrt{2}}{5}u \text{ ms}^{-1}}$$

(c) the assumption that the sphere is smooth means the impulse only acts along the centres